
CHAPTER 4

DISCOUNTED CASH FLOW VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. Assuming positive cash flows and interest rates, the future value increases and the present value decreases.
2. Assuming positive cash flows and interest rates, the present value will fall and the future value will rise.
3. The better deal is the one with equal installments.
4. Yes, they should. APRs generally don't provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.
5. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.
6. It's a reflection of the time value of money. TMCC gets to use the \$24,099 immediately. If TMCC uses it wisely, it will be worth more than \$100,000 in thirty years.
7. This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a "call" feature. Such features are discussed at length in a later chapter.
8. The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the \$100,000? Thus, our answer does depend on who is making the promise to repay.
9. The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.
10. The price would be higher because, as time passes, the price of the security will tend to rise toward \$100,000. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the \$100,000 grows shorter, and the present value rises. In 2019, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC financial position could deteriorate. Either event would tend to depress the security's price.

Solutions to Questions and Problems

NOTE: All-end-of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The simple interest per year is:

$$\$5,000 \times .09 = \$450$$

So, after 10 years, you will have:

$$\$450 \times 10 = \$4,500 \text{ in interest.}$$

The total balance will be $\$5,000 + 4,500 = \$9,500$

With compound interest, we use the future value formula:

$$FV = PV(1 + r)^t$$

$$FV = \$5,000(1.09)^{10} = \$11,836.82$$

The difference is:

$$\$11,836.82 - 9,500 = \$2,336.82$$

2. To find the FV of a lump sum, we use:

$$FV = PV(1 + r)^t$$

$$a. \quad FV = \$1,000(1.06)^{10} = \$1,790.85$$

$$b. \quad FV = \$1,000(1.09)^{10} = \$2,367.36$$

$$c. \quad FV = \$1,000(1.06)^{20} = \$3,207.14$$

- d. Because interest compounds on the interest already earned, the interest earned in part c is more than twice the interest earned in part a. With compound interest, future values grow exponentially.

3. To find the PV of a lump sum, we use:

$$PV = FV / (1 + r)^t$$

$$PV = \$15,451 / (1.07)^6 = \$10,295.65$$

$$PV = \$51,557 / (1.15)^9 = \$14,655.72$$

$$PV = \$886,073 / (1.11)^{18} = \$135,411.60$$

$$PV = \$550,164 / (1.18)^{23} = \$12,223.79$$

4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for r , we get:

$$r = (FV / PV)^{1/t} - 1$$

$$\begin{aligned} FV = \$307 &= \$242(1 + r)^2; & r &= (\$307 / \$242)^{1/2} - 1 & = 12.63\% \\ FV = \$896 &= \$410(1 + r)^9; & r &= (\$896 / \$410)^{1/9} - 1 & = 9.07\% \\ FV = \$162,181 &= \$51,700(1 + r)^{15}; & r &= (\$162,181 / \$51,700)^{1/15} - 1 & = 7.92\% \\ FV = \$483,500 &= \$18,750(1 + r)^{30}; & r &= (\$483,500 / \$18,750)^{1/30} - 1 & = 11.44\% \end{aligned}$$

5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for t , we get:

$$t = \ln(FV / PV) / \ln(1 + r)$$

$$\begin{aligned} FV = \$1,284 &= \$625(1.06)^t; & t &= \ln(\$1,284 / \$625) / \ln 1.06 & = 12.36 \text{ years} \\ FV = \$4,341 &= \$810(1.13)^t; & t &= \ln(\$4,341 / \$810) / \ln 1.13 & = 13.74 \text{ years} \\ FV = \$402,662 &= \$18,400(1.32)^t; & t &= \ln(\$402,662 / \$18,400) / \ln 1.32 & = 11.11 \text{ years} \\ FV = \$173,439 &= \$21,500(1.16)^t; & t &= \ln(\$173,439 / \$21,500) / \ln 1.16 & = 14.07 \text{ years} \end{aligned}$$

6. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for t , we get:

$$t = \ln(FV / PV) / \ln(1 + r)$$

The length of time to double your money is:

$$\begin{aligned} FV &= \$2 = \$1(1.09)^t \\ t &= \ln 2 / \ln 1.09 = 8.04 \text{ years} \end{aligned}$$

The length of time to quadruple your money is:

$$\begin{aligned} FV &= \$4 = \$1(1.09)^t \\ t &= \ln 4 / \ln 1.09 = 16.09 \text{ years} \end{aligned}$$

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

7. To find the PV of a lump sum, we use:

$$PV = FV / (1 + r)^t$$

$$PV = \$750,000,000 / (1.082)^{20} = \$155,065,808.54$$

8. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$$FV = PV(1 + r)^t$$

Solving for r , we get:

$$r = (FV / PV)^{1/t} - 1$$

$$r = (\$10,311,500 / \$12,377,500)^{1/4} - 1 = -4.46\%$$

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

9. A consol is a perpetuity. To find the PV of a perpetuity, we use the equation:

$$PV = C / r$$

$$PV = \$120 / .057$$

$$PV = \$2,105.26$$

10. To find the future value with continuous compounding, we use the equation:

$$FV = PVe^{Rt}$$

a.	$FV = \$1,900e^{12(5)}$	$= \$3,462.03$
b.	$FV = \$1,900e^{10(3)}$	$= \$2,564.73$
c.	$FV = \$1,900e^{05(10)}$	$= \$3,132.57$
d.	$FV = \$1,900e^{07(8)}$	$= \$3,326.28$

11. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

$$PV = FV / (1 + r)^t$$

$$PV@10\% = \$1,200 / 1.10 + \$730 / 1.10^2 + \$965 / 1.10^3 + \$1,590 / 1.10^4 = \$3,505.23$$

$$PV@18\% = \$1,200 / 1.18 + \$730 / 1.18^2 + \$965 / 1.18^3 + \$1,590 / 1.18^4 = \$2,948.66$$

$$PV@24\% = \$1,200 / 1.24 + \$730 / 1.24^2 + \$965 / 1.24^3 + \$1,590 / 1.24^4 = \$2,621.17$$

12. To find the PVA, we use the equation:

$$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$$

At a 5 percent interest rate:

$$X@5\%: PVA = \$5,500\{[1 - (1/1.05)^9] / .05\} = \$39,093.02$$

$$Y@5\%: PVA = \$8,000\{[1 - (1/1.05)^5] / .05\} = \$34,635.81$$

And at a 22 percent interest rate:

$$X@22\%: PVA = \$5,500\{[1 - (1/1.22)^9] / .22\} = \$20,824.57$$

$$Y@22\%: PVA = \$8,000\{[1 - (1/1.22)^5] / .22\} = \$22,909.12$$

Notice that the PV of Cash flow X has a greater PV at a 5 percent interest rate, but a lower PV at a 22 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At a higher interest rate, these bigger cash flows early are more important since the cost of waiting (the interest rate) is so much greater.

13. To find the PVA, we use the equation:

$$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$$

$$PVA@15 \text{ yrs: } PVA = \$4,300\{[1 - (1/1.09)^{15}] / .09\} = \$34,660.96$$

$$PVA@40 \text{ yrs: } PVA = \$4,300\{[1 - (1/1.09)^{40}] / .09\} = \$46,256.65$$

$$PVA@75 \text{ yrs: } PVA = \$4,300\{[1 - (1/1.09)^{75}] / .09\} = \$47,703.26$$

To find the PV of a perpetuity, we use the equation:

$$PV = C / r$$

$$PV = \$4,300 / .09$$

$$PV = \$47,777.78$$

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only \$74.51.

14. This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

$$PV = C / r$$

$$PV = \$20,000 / .065 = \$307,692.31$$

To find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

$$PV = C / r$$

$$\$340,000 = \$20,000 / r$$

We can now solve for the interest rate as follows:

$$r = \$20,000 / \$340,000 = .0588 \text{ or } 5.88\%$$

15. For discrete compounding, to find the EAR, we use the equation:

$$EAR = [1 + (APR / m)]^m - 1$$

$$EAR = [1 + (.08 / 4)]^4 - 1 = .0824 \text{ or } 8.24\%$$

$$EAR = [1 + (.18 / 12)]^{12} - 1 = .1956 \text{ or } 19.56\%$$

$$EAR = [1 + (.12 / 365)]^{365} - 1 = .1275 \text{ or } 12.75\%$$

To find the EAR with continuous compounding, we use the equation:

$$EAR = e^q - 1$$

$$EAR = e^{.14} - 1 = .1503 \text{ or } 15.03\%$$

16. Here, we are given the EAR and need to find the APR. Using the equation for discrete compounding:

$$EAR = [1 + (APR / m)]^m - 1$$

We can now solve for the APR. Doing so, we get:

$$APR = m[(1 + EAR)^{1/m} - 1]$$

$$EAR = .1030 = [1 + (APR / 2)]^2 - 1 \quad APR = 2[(1.1030)^{1/2} - 1] = .1005 \text{ or } 10.05\%$$

$$EAR = .0940 = [1 + (APR / 12)]^{12} - 1 \quad APR = 12[(1.0940)^{1/12} - 1] = .0902 \text{ or } 9.02\%$$

$$EAR = .0720 = [1 + (APR / 52)]^{52} - 1 \quad APR = 52[(1.0720)^{1/52} - 1] = .0696 \text{ or } 6.96\%$$

Solving the continuous compounding EAR equation:

$$EAR = e^q - 1$$

We get:

$$APR = \ln(1 + EAR)$$

$$APR = \ln(1 + .1590)$$

$$APR = .1476 \text{ or } 14.76\%$$

17. For discrete compounding, to find the EAR, we use the equation:

$$\text{EAR} = [1 + (\text{APR} / m)]^m - 1$$

So, for each bank, the EAR is:

$$\text{First National: } \text{EAR} = [1 + (.1010 / 12)]^{12} - 1 = .1058 \text{ or } 10.58\%$$

$$\text{First United: } \text{EAR} = [1 + (.1040 / 2)]^2 - 1 = .1067 \text{ or } 10.67\%$$

A higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.

18. The cost of a case of wine is 10 percent less than the cost of 12 individual bottles, so the cost of a case will be:

$$\text{Cost of case} = (12)(\$10)(1 - .10)$$

$$\text{Cost of case} = \$108$$

Now, we need to find the interest rate. The cash flows are an annuity due, so:

$$\text{PVA} = (1 + r) C(\{1 - [1/(1 + r)]^t\} / r)$$

$$\$108 = (1 + r) \$10(\{1 - [1 / (1 + r)^{12}] / r)$$

Solving for the interest rate, we get:

$$r = .0198 \text{ or } 1.98\% \text{ per week}$$

So, the APR of this investment is:

$$\text{APR} = .0198(52)$$

$$\text{APR} = 1.0277 \text{ or } 102.77\%$$

And the EAR is:

$$\text{EAR} = (1 + .0198)^{52} - 1$$

$$\text{EAR} = 1.7668 \text{ or } 176.68\%$$

The analysis appears to be correct. He really can earn about 177 percent buying wine by the case. The only question left is this: Can you really find a fine bottle of Bordeaux for \$10?

19. Here, we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

$$\text{PVA} = C(\{1 - [1/(1 + r)]^t\} / r)$$

$$\$18,400 = \$600 \{ [1 - (1/1.009)^t] / .009 \}$$

Now, we solve for t :

$$1/1.009^t = 1 - [(\$18,400)(.009) / (\$600)]$$

$$1.009^t = 1/(0.724) = 1.381$$

$$t = \ln 1.381 / \ln 1.009 = 36.05 \text{ months}$$

20. Here, we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

$$FV = PV(1 + r)$$

$$\$4 = \$3(1 + r)$$

$$r = 4/3 - 1 = 33.33\% \text{ per week}$$

The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

$$APR = (52)33.33\% = 1,733.33\%$$

And using the equation to find the EAR:

$$EAR = [1 + (APR / m)]^m - 1$$

$$EAR = [1 + .3333]^{52} - 1 = 313,916,515.69\%$$

Intermediate

21. To find the FV of a lump sum with discrete compounding, we use:

$$FV = PV(1 + r)^t$$

$$a. \quad FV = \$1,000(1.08)^7 = \$1,713.82$$

$$b. \quad FV = \$1,000(1 + .08/2)^{14} = \$1,731.68$$

$$c. \quad FV = \$1,000(1 + .08/12)^{84} = \$1,747.42$$

- d. To find the future value with continuous compounding, we use the equation:

$$FV = PVe^{Rt}$$

$$FV = \$1,000e^{.08(7)} = \$1,750.67$$

- e. The future value increases when the compounding period is shorter because interest is earned on previously accrued interest. The shorter the compounding period, the more frequently interest is earned, and the greater the future value, assuming the same stated interest rate.

22. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

$$.06(10) = .6$$

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of \$1, or:

$$(1 + r)^{10}$$

Setting the two equal, we get:

$$(.06)(10) = (1 + r)^{10} - 1$$

$$r = 1.6^{1/10} - 1 = .0481 \text{ or } 4.81\%$$

23. We need to find the annuity payment in retirement. Our retirement savings ends at the same time the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

$$\text{Stock account: FVA} = \$700[\{[1 + (.10/12)]^{360} - 1\} / (.10/12)] = \$1,582,341.55$$

$$\text{Bond account: FVA} = \$300[\{[1 + (.06/12)]^{360} - 1\} / (.06/12)] = \$301,354.51$$

So, the total amount saved at retirement is:

$$\$1,582,341.55 + 301,354.51 = \$1,883,696.06$$

Solving for the withdrawal amount in retirement using the PVA equation gives us:

$$\begin{aligned} \text{PVA} &= \$1,883,696.06 = C[1 - \{1 / [1 + (.08/12)]^{300}\} / (.08/12)] \\ C &= \$1,883,696.06 / 129.5645 = \$14,538.67 \text{ withdrawal per month} \end{aligned}$$

24. Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is four times as large as the PV. The number of periods is four, the number of quarters per year. So:

$$\begin{aligned} \text{FV} &= \$4 = \$1(1 + r)^{(12/3)} \\ r &= .4142 \text{ or } 41.42\% \end{aligned}$$

25. Here, we need to find the interest rate for two possible investments. Each investment is a lump sum, so:

$$\begin{aligned} \text{G: } \text{PV} &= \$75,000 = \$135,000 / (1 + r)^6 \\ (1 + r)^6 &= \$135,000 / \$75,000 \\ r &= (1.80)^{1/6} - 1 = .1029 \text{ or } 10.29\% \end{aligned}$$

$$\begin{aligned} \text{H: } \text{PV} &= \$75,000 = \$195,000 / (1 + r)^{10} \\ (1 + r)^{10} &= \$195,000 / \$75,000 \\ r &= (2.60)^{1/10} - 1 = .1003 \text{ or } 10.03\% \end{aligned}$$

26. This is a growing perpetuity. The present value of a growing perpetuity is:

$$\begin{aligned}PV &= C / (r - g) \\PV &= \$215,000 / (.10 - .04) \\PV &= \$3,583,333.33\end{aligned}$$

It is important to recognize that when dealing with annuities or perpetuities, the present value equation calculates the present value one period before the first payment. In this case, since the first payment is in two years, we have calculated the present value one year from now. To find the value today, we simply discount this value as a lump sum. Doing so, we find the value of the cash flow stream today is:

$$\begin{aligned}PV &= FV / (1 + r)^t \\PV &= \$3,583,333.33 / (1 + .10)^1 \\PV &= \$3,257,575.76\end{aligned}$$

27. The dividend payments are made quarterly, so we must use the quarterly interest rate. The quarterly interest rate is:

$$\begin{aligned}\text{Quarterly rate} &= \text{Stated rate} / 4 \\ \text{Quarterly rate} &= .07 / 4 \\ \text{Quarterly rate} &= .0175\end{aligned}$$

Using the present value equation for a perpetuity, we find the value today of the dividends paid must be:

$$\begin{aligned}PV &= C / r \\PV &= \$5 / .0175 \\PV &= \$285.71\end{aligned}$$

28. We can use the PVA annuity equation to answer this question. The annuity has 23 payments, not 22 payments. Since there is a payment made in Year 3, the annuity actually begins in Year 2. So, the value of the annuity in Year 2 is:

$$\begin{aligned}PVA &= C(\{1 - [1/(1 + r)]^t\} / r) \\PVA &= \$5,000(\{1 - [1/(1 + .08)]^{23}\} / .08) \\PVA &= \$51,855.29\end{aligned}$$

This is the value of the annuity one period before the first payment, or Year 2. So, the value of the cash flows today is:

$$\begin{aligned}PV &= FV/(1 + r)^t \\PV &= \$51,855.29 / (1 + .08)^2 \\PV &= \$44,457.56\end{aligned}$$

29. We need to find the present value of an annuity. Using the PVA equation, and the 15 percent interest rate, we get:

$$\begin{aligned}PVA &= C(\{1 - [1/(1 + r)]^t\} / r) \\PVA &= \$750(\{1 - [1/(1 + .15)]^{15}\} / .15) \\PVA &= \$4,385.53\end{aligned}$$

This is the value of the annuity in Year 5, one period before the first payment. Finding the value of this amount today, we find:

$$\begin{aligned} PV &= FV / (1 + r)^t \\ PV &= \$4,385.53 / (1 + .12)^5 \\ PV &= \$2,488.47 \end{aligned}$$

- 30.** The amount borrowed is the value of the home times one minus the down payment, or:

$$\begin{aligned} \text{Amount borrowed} &= \$450,000(1 - .20) \\ \text{Amount borrowed} &= \$360,000 \end{aligned}$$

The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

$$\begin{aligned} PVA &= \$360,000 = C(\{1 - [1 / (1 + .075/12)]^{360}\} / (.075/12)) \\ C &= \$2,517.17 \end{aligned}$$

Now, at time = 8, we need to find the PV of the payments which have not been made. The balloon payment will be:

$$\begin{aligned} PVA &= \$2,517.17(\{1 - [1 / (1 + .075/12)]^{22(12)}\} / (.075/12)) \\ PVA &= \$325,001.73 \end{aligned}$$

- 31.** Here, we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

$$FV = \$6,000 [1 + (.024/12)]^6 = \$6,072.36$$

This is the balance in six months. The FV in another six months will be:

$$FV = \$6,072.36 [1 + (.18/12)]^6 = \$6,639.78$$

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

$$\text{Interest} = \$6,639.78 - 6,000 = \$639.78$$

- 32.** The company would be indifferent at the interest rate that makes the present value of the cash flows equal to the cost today. Since the cash flows are a perpetuity, we can use the PV of a perpetuity equation. Doing so, we find:

$$\begin{aligned} PV &= C / r \\ \$150,000 &= \$13,000 / r \\ r &= \$13,000 / \$150,000 \\ r &= .0867 \text{ or } 8.67\% \end{aligned}$$

33. The company will accept the project if the present value of the increased cash flows is greater than the cost. The cash flows are a growing perpetuity, so the present value is:

$$\begin{aligned} PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ PV &= \$18,000 \{ [1/ (.11 - .04)] - [1/ (.11 - .04)] \times [(1 + .04)/(1 + .11)]^5 \} \\ PV &= \$71,479.47 \end{aligned}$$

The company should accept the project since the cost is less than the increased cash flows.

34. Since your salary grows at 4 percent per year, your salary next year will be:

$$\begin{aligned} \text{Next year's salary} &= \$60,000 (1 + .04) \\ \text{Next year's salary} &= \$62,400 \end{aligned}$$

This means your deposit next year will be:

$$\begin{aligned} \text{Next year's deposit} &= \$62,400(.05) \\ \text{Next year's deposit} &= \$3,120 \end{aligned}$$

Since your salary grows at 4 percent, your deposit will also grow at 4 percent. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

$$\begin{aligned} PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ PV &= \$3,120 \{ [1/ (.09 - .04)] - [1/ (.09 - .04)] \times [(1 + .04)/(1 + .09)]^{40} \} \\ PV &= \$52,861.98 \end{aligned}$$

Now, we can find the future value of this lump sum in 40 years. We find:

$$\begin{aligned} FV &= PV(1 + r)^t \\ FV &= \$52,861.98(1 + .09)^{40} \\ FV &= \$1,660,364.12 \end{aligned}$$

This is the value of your savings in 40 years.

35. The relationship between the PVA and the interest rate is:

PVA falls as r increases, and PVA rises as r decreases
FVA rises as r increases, and FVA falls as r decreases

The present values of \$7,500 per year for 12 years at the various interest rates given are:

$$PVA@10\% = \$7,500 \{ [1 - (1/1.10)^{12}] / .10 \} = \$51,102.69$$

$$PVA@5\% = \$7,500 \{ [1 - (1/1.05)^{12}] / .05 \} = \$66,474.39$$

$$PVA@15\% = \$7,500 \{ [1 - (1/1.15)^{12}] / .15 \} = \$40,654.64$$

36. Here, we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

$$FVA = \$30,000 = \$250[\{[1 + (.10/12)]^t - 1\} / (.10/12)]$$

Solving for t , we get:

$$1.00833^t = 1 + [(\$30,000)(.10/12) / \$250]$$

$$t = \ln 2 / \ln 1.00833 = 83.52 \text{ payments}$$

37. Here, we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

$$PVA = \$80,000 = \$1,650[1 - [1 / (1 + r)]^{60}] / r]$$

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

$$r = 0.727\%$$

The APR is the periodic interest rate times the number of periods in the year, so:

$$APR = 12(0.727\%) = 8.72\%$$

38. The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the \$1,200 monthly payments is:

$$PVA = \$1,200[(1 - \{1 / [1 + (.068/12)]\}^{360}) / (.068/12)] = \$184,070.20$$

The monthly payments of \$1,200 will amount to a principal payment of \$184,070.20. The amount of principal you will still owe is:

$$\$250,000 - 184,070.20 = \$65,929.80$$

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

$$\text{Balloon payment} = \$65,929.80[1 + (.068/12)]^{360} = \$504,129.05$$

39. We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know are:

$$\text{PV of Year 1 CF: } \$1,200 / 1.10 = \$1,090.91$$

$$\text{PV of Year 3 CF: } \$2,400 / 1.10^3 = \$1,803.16$$

$$\text{PV of Year 4 CF: } \$2,600 / 1.10^4 = \$1,775.83$$

So, the PV of the missing CF is:

$$\$6,453 - 1,090.91 - 1,803.16 - 1,775.83 = \$1,783.10$$

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

$$\$1,783.10(1.10)^2 = \$2,157.55$$

40. To solve this problem, we simply need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of \$1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

$$\begin{aligned} & \$1,000,000 + \$1,350,000/1.09 + \$1,700,000/1.09^2 + \$2,050,000/1.09^3 + \$2,400,000/1.09^4 + \\ & \$2,750,000/1.09^5 + \$3,100,000/1.09^6 + \$3,450,000/1.09^7 + \$3,800,000/1.09^8 + \\ & \$4,150,000/1.09^9 + \$4,500,000/1.09^{10} = \$18,194,308.69 \end{aligned}$$

41. Here, we are finding interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We need to solve for the number of payments. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

$$\text{Amount borrowed} = 0.80(\$2,600,000) = \$2,080,000$$

Using the PVA equation:

$$\text{PVA} = \$2,080,000 = \$14,000[\{ 1 - [1 / (1 + r)]^{360} \} / r]$$

Unfortunately, this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

$$r = 0.593\%$$

The APR is the monthly interest rate times the number of months in the year, so:

$$\text{APR} = 12(0.593\%) = 7.12\%$$

And the EAR is:

$$\text{EAR} = (1 + .00593)^{12} - 1 = .0735 \text{ or } 7.35\%$$

42. The profit the firm earns is just the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

$$\text{PV} = \$135,000 / 1.13^3 = \$93,561.77$$

And the firm's profit is:

$$\text{Profit} = \$93,561.77 - 96,000.00 = -\$2,438.23$$

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

$$\begin{aligned} \$96,000 &= \$135,000 / (1 + r)^3 \\ r &= (\$135,000 / \$96,000)^{1/3} - 1 = .1204 \text{ or } 12.04\% \end{aligned}$$

43. We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 17 payments, so the PV of the annuity is:

$$\text{PVA} = \$4,000 \{ [1 - (1/1.07)^{17}] / .07 \} = \$39,052.89$$

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at $t = 8$. To find the value today, we find the PV of this lump sum. The value today is:

$$\text{PV} = \$39,052.89 / 1.07^8 = \$22,729.14$$

44. This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

$$\text{PVA}_2 = \$1,500 \{ [1 - 1 / [1 + (.09/12)]^{96}] / (.09/12) \} = \$102,387.66$$

Note that this is the PV of this annuity exactly seven years from today. Now, we can discount this lump sum to today. The value of this cash flow today is:

$$\text{PV} = \$102,387.66 / [1 + (.13/12)]^{84} = \$41,415.70$$

Now, we need to find the PV of the annuity for the first seven years. The value of these cash flows today is:

$$\text{PVA}_1 = \$1,500 \{ [1 - 1 / [1 + (.13/12)]^{84}] / (.13/12) \} = \$82,453.99$$

The value of the cash flows today is the sum of these two cash flows, so:

$$\text{PV} = \$82,453.99 + 41,415.70 = \$123,869.99$$

45. Here, we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate, and payments. First, we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

$$\text{FVA} = \$1,200 \{ [1 + (.098/12)]^{180} - 1 \} / (.098/12) = \$488,328.61$$

Now, we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

$$\begin{aligned} \text{FV} &= \$488,328.61 = \text{PV} e^{.09(15)} \\ \text{PV} &= \$488,328.61 e^{-1.35} = \$126,594.44 \end{aligned}$$

46. To find the value of the perpetuity at $t = 7$, we first need to use the PV of a perpetuity equation. Using this equation we find:

$$PV = \$2,100 / .073 = \$28,767.12$$

Remember that the PV of a perpetuity (and annuity) equations give the PV one period before the first payment, so, this is the value of the perpetuity at $t = 14$. To find the value at $t = 7$, we find the PV of this lump sum as:

$$PV = \$28,767.12 / 1.073^7 = \$17,567.03$$

47. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

$$PVA = \$26,000 = \$2,491.67 \{ (1 - [1 / (1 + r)]^{12}) / r \}$$

Again, we cannot solve this equation for r , so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

$$r = 2.219\% \text{ per month}$$

So the APR is:

$$APR = 12(2.219\%) = 26.62\%$$

And the EAR is:

$$EAR = (1.02219)^{12} - 1 = 30.12\%$$

48. The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

$$\text{Monthly rate} = .12 / 12 = .01$$

To get the semiannual interest rate, we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

$$\text{Semiannual rate} = (1.01)^6 - 1 = 6.15\%$$

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

$$PVA @ t = 9: \$4,500 \{ [1 - (1 / 1.0615)^{10}] / .0615 \} = \$32,883.16$$

Note, that this is the value one period (six months) before the first payment, so it is the value at $t = 9$. So, the value at the various times the questions asked for uses this value 9 years from now.

$$PV @ t = 5: \$32,883.16 / 1.0615^8 = \$20,396.12$$

Note, that you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

$$\text{EAR} = (1 + .01)^{12} - 1 = 12.68\%$$

So, we can find the PV at $t = 5$ using the following method as well:

$$\text{PV @ } t = 5: \$32,883.16 / 1.1268^4 = \$20,396.12$$

The value of the annuity at the other times in the problem is:

$$\text{PV @ } t = 3: \$32,883.16 / 1.0615^{12} = \$16,063.29$$

$$\text{PV @ } t = 3: \$32,883.16 / 1.1268^6 = \$16,063.29$$

$$\text{PV @ } t = 0: \$32,883.16 / 1.0615^{18} = \$11,227.04$$

$$\text{PV @ } t = 0: \$32,883.16 / 1.1268^9 = \$11,227.04$$

49. a. If the payments are in the form of an ordinary annuity, the present value will be:

$$\text{PVA} = C \{ [1 - 1/(1 + r)^t] / r \}$$

$$\text{PVA} = \$10,000 \{ [1 - 1/(1 + .11)^5] / .11 \}$$

$$\text{PVA} = \$36,958.97$$

If the payments are an annuity due, the present value will be:

$$\text{PVA}_{\text{due}} = (1 + r) \text{PVA}$$

$$\text{PVA}_{\text{due}} = (1 + .11) \$36,958.97$$

$$\text{PVA}_{\text{due}} = \$41,024.46$$

- b. We can find the future value of the ordinary annuity as:

$$\text{FVA} = C \{ [(1 + r)^t - 1] / r \}$$

$$\text{FVA} = \$10,000 \{ [(1 + .11)^5 - 1] / .11 \}$$

$$\text{FVA} = \$62,278.01$$

If the payments are an annuity due, the future value will be:

$$\text{FVA}_{\text{due}} = (1 + r) \text{FVA}$$

$$\text{FVA}_{\text{due}} = (1 + .11) \$62,278.01$$

$$\text{FVA}_{\text{due}} = \$69,128.60$$

- c. Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary due will always higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

50. We need to use the PVA due equation, that is:

$$PVA_{\text{due}} = (1 + r) PVA$$

Using this equation:

$$PVA_{\text{due}} = \$65,000 = [1 + (.0645/12)] \times C \{ [1 - 1 / [1 + (.0645/12)]^{48}] / (.0645/12) \}$$

$$C = \$1,531.74$$

Notice, to find the payment for the PVA due we simply compound the payment for an ordinary annuity forward one period.

Challenge

51. The monthly interest rate is the annual interest rate divided by 12, or:

$$\text{Monthly interest rate} = .104 / 12$$

$$\text{Monthly interest rate} = .00867$$

Now we can set the present value of the lease payments equal to the cost of the equipment, or \$3,500. The lease payments are in the form of an annuity due, so:

$$PVA_{\text{due}} = (1 + r) C \{ [1 - 1/(1 + r)^t] / r \}$$

$$\$3,500 = (1 + .00867) C \{ [1 - 1/(1 + .00867)^{24}] / .00867 \}$$

$$C = \$160.76$$

52. First, we will calculate the present value of the college expenses for each child. The expenses are an annuity, so the present value of the college expenses is:

$$PVA = C \{ [1 - 1/(1 + r)^t] / r \}$$

$$PVA = \$35,000 \{ [1 - 1/(1 + .085)^4] / .085 \}$$

$$PVA = \$114,645.88$$

This is the cost of each child's college expenses one year before they enter college. So, the cost of the oldest child's college expenses today will be:

$$PV = FV / (1 + r)^t$$

$$PV = \$114,645.88 / (1 + .085)^{14}$$

$$PV = \$36,588.29$$

And the cost of the youngest child's college expenses today will be:

$$PV = FV / (1 + r)^t$$

$$PV = \$114,645.88 / (1 + .085)^{16}$$

$$PV = \$31,080.12$$

Therefore, the total cost today of your children's college expenses is:

$$\text{Cost today} = \$36,588.29 + 31,080.12$$

$$\text{Cost today} = \$67,668.41$$

This is the present value of your annual savings, which are an annuity. So, the amount you must save each year will be:

$$\begin{aligned} PVA &= C(\{1 - [1/(1 + r)]^t\} / r) \\ \$67,668.41 &= C(\{1 - [1/(1 + .085)]^{15}\} / .085) \\ C &= \$8,148.66 \end{aligned}$$

53. The salary is a growing annuity, so using the equation for the present value of a growing annuity. The salary growth rate is 3.5 percent and the discount rate is 12 percent, so the value of the salary offer today is:

$$\begin{aligned} PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ PV &= \$45,000 \{ [1/(.12 - .035)] - [1/(.12 - .035)] \times [(1 + .035)/(1 + .12)]^{25} \} \\ PV &= \$455,816.18 \end{aligned}$$

The yearly bonuses are 10 percent of the annual salary. This means that next year's bonus will be:

$$\begin{aligned} \text{Next year's bonus} &= .10(\$45,000) \\ \text{Next year's bonus} &= \$4,500 \end{aligned}$$

Since the salary grows at 3.5 percent, the bonus will grow at 3.5 percent as well. Using the growing annuity equation, with a 3.5 percent growth rate and a 12 percent discount rate, the present value of the annual bonuses is:

$$\begin{aligned} PV &= C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \} \\ PV &= \$4,500 \{ [1/(.12 - .035)] - [1/(.12 - .035)] \times [(1 + .035)/(1 + .12)]^{25} \} \\ PV &= \$45,581.62 \end{aligned}$$

Notice the present value of the bonus is 10 percent of the present value of the salary. The present value of the bonus will always be the same percentage of the present value of the salary as the bonus percentage. So, the total value of the offer is:

$$\begin{aligned} PV &= PV(\text{Salary}) + PV(\text{Bonus}) + \text{Bonus paid today} \\ PV &= \$455,816.18 + 45,581.62 + 10,000 \\ PV &= \$511,397.80 \end{aligned}$$

54. Here, we need to compare to options. In order to do so, we must get the value of the two cash flow streams to the same time, so we will find the value of each today. We must also make sure to use the aftertax cash flows, since it is more relevant. For Option A, the aftertax cash flows are:

$$\begin{aligned} \text{Aftertax cash flows} &= \text{Pretax cash flows}(1 - \text{tax rate}) \\ \text{Aftertax cash flows} &= \$175,000(1 - .28) \\ \text{Aftertax cash flows} &= \$126,000 \end{aligned}$$

The aftertax cash flows from Option A are in the form of an annuity due, so the present value of the cash flow today is:

$$\begin{aligned} PVA_{\text{due}} &= (1 + r) C(\{1 - [1/(1 + r)]^t\} / r) \\ PVA_{\text{due}} &= (1 + .10)\$126,000(\{1 - [1/(1 + .10)]^{31}\} / .10) \\ PVA_{\text{due}} &= \$1,313,791.22 \end{aligned}$$

For Option B, the aftertax cash flows are:

$$\text{Aftertax cash flows} = \text{Pretax cash flows}(1 - \text{tax rate})$$

$$\text{Aftertax cash flows} = \$125,000(1 - .28)$$

$$\text{Aftertax cash flows} = \$90,000$$

The aftertax cash flows from Option B are an ordinary annuity, plus the cash flow today, so the present value:

$$PV = C(\{1 - [1/(1 + r)]^t\} / r) + CF_0$$

$$PV = \$90,000\{1 - [1/(1 + .10)]^{30}\} / .10 + \$530,000$$

$$PV = \$1,378,422.30$$

You should choose Option B because it has a higher present value on an aftertax basis.

55. We need to find the first payment into the retirement account. The present value of the desired amount at retirement is:

$$PV = FV/(1 + r)^t$$

$$PV = \$1,500,000/(1 + .10)^{30}$$

$$PV = \$85,962.83$$

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

$$PV = C \{ [1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t \}$$

$$\$85,962.83 = C \{ [1/(\cdot 10 - \cdot 03)] - [1/(\cdot 10 - \cdot 03)] \times [(1 + \cdot 03)/(1 + \cdot 10)]^{30} \}$$

$$C = \$6,989.68$$

This is the amount you need to save next year. So, the percentage of your salary is:

$$\text{Percentage of salary} = \$6,989.68/\$70,000$$

$$\text{Percentage of salary} = .0999 \text{ or } 9.99\%$$

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

56. Since she put \$1,000 down, the amount borrowed will be:

$$\text{Amount borrowed} = \$25,000 - 1,000$$

$$\text{Amount borrowed} = \$24,000$$

So, the monthly payments will be:

$$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$$

$$\$24,000 = C[1 - [1/(1 + \cdot 084/12)]^{60}] / (\cdot 084/12)$$

$$C = \$491.24$$

The amount remaining on the loan is the present value of the remaining payments. Since the first payment was made on October 1, 2007, and she made a payment on October 1, 2009, there are 35 payments remaining, with the first payment due immediately. So, we can find the present value of the remaining 34 payments after November 1, 2009, and add the payment made on this date. So the remaining principal owed on the loan is:

$$\begin{aligned} PV &= C(\{1 - [1/(1 + r)]^t\} / r) + C_0 \\ PV &= \$491.24[\{1 - [1/(1 + .084/12)]^{34}\} / (.084/12)] \\ C &= \$14,817.47 \end{aligned}$$

She must also pay a one percent prepayment penalty and the payment due on November 1, 2009, so the total amount of the payment is:

$$\begin{aligned} \text{Total payment} &= \text{Balloon amount}(1 + \text{Prepayment penalty}) + \text{Current payment} \\ \text{Total payment} &= \$14,817.47(1 + .01) + \$491.24 \\ \text{Total payment} &= \$15,456.89 \end{aligned}$$

57. The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

$$\text{EAR} = .11 = [1 + (\text{APR} / 12)]^{12} - 1; \quad \text{APR} = 12[(1.11)^{1/12} - 1] = 10.48\%$$

And the post-retirement APR is:

$$\text{EAR} = .08 = [1 + (\text{APR} / 12)]^{12} - 1; \quad \text{APR} = 12[(1.08)^{1/12} - 1] = 7.72\%$$

First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

$$\text{PVA} = \$20,000\{1 - [1 / (1 + .0772/12)]^{12(20)}\} / (.0772/12) = \$2,441,554.61$$

$$\text{PV} = \$1,000,000 / (1 + .08)^{20} = \$214,548.21$$

So, at retirement, he needs:

$$\$2,441,554.61 + 214,548.21 = \$2,656,102.81$$

He will be saving \$1,900 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

$$\text{FVA} = \$1,900[\{1 + (.1048/12)\}^{12(10)} - 1] / (.1048/12) = \$400,121.62$$

After he purchases the cabin, the amount he will have left is:

$$\$400,121.62 - 320,000 = \$80,121.62$$

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

$$\text{FV} = \$80,121.62[1 + (.1048/12)]^{12(20)} = \$646,965.50$$

So, when he is ready to retire, based on his current savings, he will be short:

$$\$2,656,102.81 - 645,965.50 = \$2,010,137.31$$

This amount is the FV of the monthly savings he must make between years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

$$\begin{aligned} FVA &= \$2,010,137.31 = C \left[\frac{1 + (.1048/12)^{12(20)} - 1}{(.1048/12)} \right] \\ C &= \$2,486.12 \end{aligned}$$

58. To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing is simply the PV of the lease payments, plus the \$1. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

$$PV = \$1 + \$520 \left\{ 1 - \left[1 / (1 + .08/12)^{12(3)} \right] \right\} / (.08/12) = \$16,595.14$$

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

$$PV = \$26,000 / [1 + (.08/12)^{12(3)}] = \$20,468.62$$

The PV of the decision to purchase is:

$$\$38,000 - 20,468.62 = \$17,531.38$$

In this case, it is cheaper to lease the car than buy it since the PV of the leasing cash flows is lower. To find the breakeven resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

$$\begin{aligned} \$38,000 - \text{PV of resale price} &= \$16,595.14 \\ \text{PV of resale price} &= \$21,404.86 \end{aligned}$$

The resale price that would make the PV of the lease versus buy decision is the FV of this value, so:

$$\text{Breakeven resale price} = \$21,404.86 [1 + (.08/12)^{12(3)}] = \$27,189.25$$

59. To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

$$EAR = [1 + (.05/365)]^{365} - 1 = 5.13\%$$

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

$$\begin{aligned} PV &= \$7,500,000 + \$4,200,000/1.0513 + \$5,100,000/1.0513^2 + \$5,900,000/1.0513^3 \\ &\quad + \$6,800,000/1.0513^4 + \$7,400,000/1.0513^5 + \$8,100,000/1.0513^6 \\ PV &= \$38,519,529.66 \end{aligned}$$

The player wants the contract increased in value by \$1,000,000, so the PV of the new contract will be:

$$PV = \$38,519,529.66 + 750,000 = \$39,269,529.66$$

The player has also requested a signing bonus payable today in the amount of \$10 million. We can simply subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

$$\$39,269,529.66 - 10,000,000 = \$29,269,529.66$$

To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter ($365 / 4$). The effective quarterly rate is:

$$\text{Effective quarterly rate} = [1 + (.05/365)]^{91.25} - 1 = .01258 \text{ or } 1.258\%$$

Now, we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

$$\begin{aligned} PVA &= \$29,269,529.66 = C\{[1 - (1/.01258)^{24}] / .01258\} \\ C &= \$1,420,476.43 \end{aligned}$$

60. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the \$20,000 you must repay in one year, and the \$17,200 you borrow today. The interest rate of the loan is:

$$\begin{aligned} \$20,000 &= \$17,200(1 + r) \\ r &= (\$20,000 / \$17,200) - 1 = .1628 \text{ or } 16.28\% \end{aligned}$$

Because of the discount, you only get the use of \$17,200, and the interest you pay on that amount is 16.28%, not 14%.

61. Here, we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate, so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

$$APR = 12[(1.09)^{1/12} - 1] = 8.65\%$$

To find the value today of the back pay from two years ago, we will find the FV of the annuity (salary), and then find the FV of the lump sum value of the salary. Doing so gives us:

$$FV = (\$42,000/12) [(1 + (.0865/12))^{24} - 1] / (.0865/12) (1 + .09) = \$47,639.05$$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way. Now, we need to find the value today of last year's back pay:

$$FVA = (\$45,000/12) \left[\frac{[1 + (.0865/12)]^{12} - 1}{(.0865/12)} \right] = \$46,827.37$$

Next, we find the value today of the five year's future salary:

$$PVA = (\$49,000/12) \left\{ \left[1 - \frac{1}{[1 + (.0865/12)]^{12(5)}} \right] / (.0865/12) \right\} = \$198,332.55$$

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

$$\begin{aligned} \text{Award} &= \$47,639.05 + 46,827.37 + 198,332.55 + 150,000 + 25,000 \\ \text{Award} &= \$467,798.97 \end{aligned}$$

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

62. Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

$$\text{Loan repayment amount} = \$10,000(1.09) = \$10,900$$

The amount you will receive today is the principal amount of the loan times one minus the points.

$$\text{Amount received} = \$10,000(1 - .03) = \$9,700$$

Now, we simply find the interest rate for this PV and FV.

$$\begin{aligned} \$10,900 &= \$9,700(1 + r) \\ r &= (\$10,900 / \$9,700) - 1 = .1237 \text{ or } 12.37\% \end{aligned}$$

With a 12 percent quoted interest rate loan and two points, the EAR is:

$$\text{Loan repayment amount} = \$10,000(1.12) = \$11,200$$

$$\text{Amount received} = \$10,000(1 - .02) = \$9,800$$

$$\begin{aligned} \$11,200 &= \$9,800(1 + r) \\ r &= (\$11,200 / \$9,800) - 1 = .1429 \text{ or } 14.29\% \end{aligned}$$

The effective rate is not affected by the loan amount, since it drops out when solving for r .

63. First, we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the \$2,100 application fee, you will need to borrow \$202,100 to have \$200,000 after deducting the fee. Solving for the payment under these circumstances, we get:

$$PVA = \$202,100 = C \{ [1 - 1/(1.00567)^{360}] / .00567 \} \text{ where } .00567 = .068/12$$

$$C = \$1,317.54$$

We can now use this amount in the PVA equation with the original amount we wished to borrow, \$200,000. Solving for r , we find:

$$PVA = \$200,000 = \$1,317.54 [\{ 1 - [1 / (1 + r)]^{360} \} / r]$$

Solving for r with a spreadsheet, on a financial calculator, or by trial and error, gives:

$$r = 0.5752\% \text{ per month}$$

$$APR = 12(0.5752\%) = 6.90\%$$

$$EAR = (1 + .005752)^{12} - 1 = .0713 \text{ or } 7.13\%$$

With the nonrefundable fee, the APR of the loan is simply the quoted APR since the fee is not considered part of the loan. So:

$$APR = 6.80\%$$

$$EAR = [1 + (.068/12)]^{12} - 1 = .0702 \text{ or } 7.02\%$$

64. Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is \$1,000, and the payments are \$43.36 per month for three years, so the interest rate on the loan is:

$$PVA = \$1,000 = \$43.36 [\{ 1 - [1 / (1 + r)]^{36} \} / r]$$

Solving for r with a spreadsheet, on a financial calculator, or by trial and error, gives:

$$r = 2.64\% \text{ per month}$$

$$APR = 12(2.64\%) = 31.65\%$$

$$EAR = (1 + .0264)^{12} - 1 = .3667 \text{ or } 36.67\%$$

It's called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

65. Here, we are solving a two-step time value of money problem. Each question asks for a different possible cash flow to fund the same retirement plan. Each savings possibility has the same FV, that is, the PV of the retirement spending when your friend is ready to retire. The amount needed when your friend is ready to retire is:

$$PVA = \$110,000 \{ [1 - (1/1.09)^{25}] / .09 \} = \$1,080,483.76$$

This amount is the same for all three parts of this question.

- a. If your friend makes equal annual deposits into the account, this is an annuity with the FVA equal to the amount needed in retirement. The required savings each year will be:

$$\begin{aligned} FVA &= \$1,080,483.76 = C[(1.09^{30} - 1) / .09] \\ C &= \$7,926.81 \end{aligned}$$

- b. Here we need to find a lump sum savings amount. Using the FV for a lump sum equation, we get:

$$\begin{aligned} FV &= \$1,080,483.76 = PV(1.09)^{30} \\ PV &= \$81,437.29 \end{aligned}$$

- c. In this problem, we have a lump sum savings in addition to an annual deposit. Since we already know the value needed at retirement, we can subtract the value of the lump sum savings at retirement to find out how much your friend is short. Doing so gives us:

$$FV \text{ of trust fund deposit} = \$50,000(1.09)^{10} = \$118,368.18$$

So, the amount your friend still needs at retirement is:

$$FV = \$1,080,483.76 - \$118,368.18 = \$962,115.58$$

Using the FVA equation, and solving for the payment, we get:

$$\$962,115.58 = C[(1.09^{30} - 1) / .09]$$

$$C = \$7,058.42$$

This is the total annual contribution, but your friend's employer will contribute \$1,500 per year, so your friend must contribute:

$$\text{Friend's contribution} = \$7,058.42 - \$1,500 = \$5,558.42$$

66. We will calculate the number of periods necessary to repay the balance with no fee first. We simply need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 18.6%:

$$PVA = \$9,000 = \$200 \{ [1 - (1/1.0155)^t] / .0155 \} \text{ where } .0155 = .186/12$$

Solving for t , we get:

$$\begin{aligned} t &= \ln \{ 1 / [1 - (\$9,000/\$200)(.0155)] \} / \ln(1.0155) \\ t &= \ln 3.3058 / \ln 1.0155 \\ t &= 77.74 \text{ months} \end{aligned}$$

Without fee and annual rate = 8.2%:

$$PVA = \$9,000 = \$200 \{ [1 - (1/1.006833)^t] / .006833 \} \text{ where } .006833 = .082/12$$

Solving for t , we get:

$$\begin{aligned} t &= \ln \{ 1 / [1 - (\$9,000/\$200)(.006833)] \} / \ln(1.006833) \\ t &= \ln 1.4440 / \ln 1.006833 \\ t &= 53.96 \text{ months} \end{aligned}$$

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 8.20%:

$$PVA = \$9,180 = \$200 \{ [1 - (1/1.006833)^t] / .006833 \} \text{ where } .006833 = .092/12$$

Solving for t , we get:

$$\begin{aligned} t &= \ln \{ 1 / [1 - (\$9,180/\$200)(.006833)] \} / \ln(1.006833) \\ t &= \ln 1.45698 / \ln 1.006833 \\ t &= 55.27 \text{ months} \end{aligned}$$

67. We need to find the FV of the premiums to compare with the cash payment promised at age 65. We have to find the value of the premiums at year 6 first since the interest rate changes at that time. So:

$$FV_1 = \$800(1.11)^5 = \$1,348.05$$

$$FV_2 = \$800(1.11)^4 = \$1,214.46$$

$$FV_3 = \$900(1.11)^3 = \$1,230.87$$

$$FV_4 = \$900(1.11)^2 = \$1,108.89$$

$$FV_5 = \$1,000(1.11)^1 = \$1,110.00$$

Value at year six = $\$1,348.05 + 1,214.46 + 1,230.87 + 1,108.89 + 1,110.00 + 1,000.00 = \$7,012.26$

Finding the FV of this lump sum at the child's 65th birthday:

$$FV = \$7,012.26(1.07)^{59} = \$379,752.76$$

The policy is not worth buying; the future value of the policy is \$379,752.76, but the policy contract will pay off \$350,000. The premiums are worth \$29,752.76 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

$$PV = \$800/1.11 + \$800/1.11^2 + \$900/1.11^3 + \$900/1.11^4 + \$1,000/1.11^5 + \$1,000/1.11^6 = \$3,749.04$$

And the value today of the \$350,000 at age 65 is:

$$PV = \$350,000/1.07^{59} = \$6,462.87$$

$$PV = \$6,462.87/1.11^6 = \$3,455.31$$

The premiums still have the higher cash flow. At time zero, the difference is \$293.73. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest \$293.73, the difference in the cash flows at time zero, for six years at an 11 percent interest rate, and then for 59 years at a seven percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth \$29,752.76, the difference in the cash flows at time 65!

68. Since the payments occur at six month intervals, we need to get the effective six-month interest rate. We can calculate the daily interest rate since we have an APR compounded daily, so the effective six-month interest rate is:

$$\text{Effective six-month rate} = (1 + \text{Daily rate})^{180} - 1$$

$$\text{Effective six-month rate} = (1 + .09/360)^{180} - 1$$

$$\text{Effective six-month rate} = .0460 \text{ or } 4.60\%$$

Now, we can use the PVA equation to find the present value of the semi-annual payments. Doing so, we find:

$$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$$

$$PVA = \$750,000(\{1 - [1/(1 + .0460)]^{40}\} / .0460)$$

$$PVA = \$13,602,152.32$$

This is the value six months from today, which is one period (six months) prior to the first payment. So, the value today is:

$$PV = \$13,602,152.32 / (1 + .0460)$$

$$PV = \$13,003,696.50$$

This means the total value of the lottery winnings today is:

$$\text{Value of winnings today} = \$13,003,696.50 + 2,000,000$$

$$\text{Value of winnings today} = \$15,003,696.50$$

You should not take the offer since the value of the offer is less than the present value of the payments.

69. Here, we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the savings. The PV of the college costs are:

$$\text{PVA} = \$20,000 \left[\frac{1 - [1 / (1 + r)]^4}{r} \right]$$

And the FV of the savings is:

$$\text{FVA} = \$8,000 \left\{ \frac{(1 + r)^6 - 1}{r} \right\}$$

Setting these two equations equal to each other, we get:

$$\$20,000 \left[\frac{1 - [1 / (1 + r)]^4}{r} \right] = \$8,000 \left\{ \frac{(1 + r)^6 - 1}{r} \right\}$$

Reducing the equation gives us:

$$(1 + r)^{10} - 4.00(1 + r)^4 + 40.00 = 0$$

Now, we need to find the roots of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

$$r = 10.57\%$$

70. Here, we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

$$\text{PV} = \$20,000 / r$$

And the PV of the annuity is:

$$\text{PVA} = \$35,000 \left[\frac{1 - [1 / (1 + r)]^{10}}{r} \right]$$

Setting them equal and solving for r , we get:

$$\$20,000 / r = \$35,000 \left[\frac{1 - [1 / (1 + r)]^{10}}{r} \right]$$

$$\$20,000 / \$35,000 = 1 - [1 / (1 + r)]^{10}$$

$$.5714^{1/10} = 1 / (1 + r)$$

$$r = 1 / .5714^{1/10} - 1$$

$$r = .0576 \text{ or } 5.76\%$$

71. The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

$$\text{Effective 2-year rate} = [1 + (.13/365)]^{365(2)} - 1 = 29.69\%$$

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

$$\text{PV} = \$8,500 / .2969 = \$28,632.06$$

This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

$$PV = \$28,632.06(1 + .13/365)^{365} = \$32,606.24$$

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

$$PV = \$28,632.06 / (1 + .13/365)^{2(365)} = \$22,077.81$$

72. To solve for the PVA due:

$$\begin{aligned} PVA &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^t} \\ PVA_{\text{due}} &= C + \frac{C}{(1+r)} + \dots + \frac{C}{(1+r)^{t-1}} \\ PVA_{\text{due}} &= (1+r) \left(\frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^t} \right) \\ PVA_{\text{due}} &= (1+r) PVA \end{aligned}$$

And the FVA due is:

$$\begin{aligned} FVA &= C + C(1+r) + C(1+r)^2 + \dots + C(1+r)^{t-1} \\ FVA_{\text{due}} &= C(1+r) + C(1+r)^2 + \dots + C(1+r)^t \\ FVA_{\text{due}} &= (1+r)[C + C(1+r) + \dots + C(1+r)^{t-1}] \\ FVA_{\text{due}} &= (1+r)FVA \end{aligned}$$

73. a. The APR is the interest rate per week times 52 weeks in a year, so:

$$APR = 52(9\%) = 468\%$$

$$EAR = (1 + .09)^{52} - 1 = 87.3442 \text{ or } 8,734.42\%$$

b. In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With a 9 percent discount, you would receive \$9.10 for every \$10 in principal, so the weekly interest rate would be:

$$\begin{aligned} \$10 &= \$9.10(1+r) \\ r &= (\$10 / \$9.10) - 1 = .0989 \text{ or } 9.89\% \end{aligned}$$